1. Let and be prime and let be prime as well. SFTSOC that . Since is the only even prime, and are odd. That means there is an integer such that and there is an integer such that . So . That means there is an integer such that , namely . So . So there is a positive factor of that is not or. That means cannot be prime, thus we have a contradiction ⇒⇐. Therefore , which proves the proposition.
2. Suppose . SFTSOC that or . Since then . Since then . Thus we have a contradiction ⇒⇐. Therefore , which proves the proposition.
3. 1. There are ways to select a hand of 13 cards from a standard 52 card deck.
   2. There are ways to select a hand of 13 cards from a standard 52 card deck. There are ways to select a hand of 13 cards from a standard 52 card deck with no hearts in the hand selected. So there are ways to select a hand of 13 cards from a standard 52 card deck with at least one heart in the hand selected.
   3. There are ways to select a hand of 13 cards from a standard 52 card deck. There are ways to select a hand of 13 cards from a standard 52 card deck with no hearts in the hand selected. There are ways to select a hand of 13 cards from a standard 52 card deck with no clubs in the hand selected. So there are ways to select a hand of 13 cards from a standard 52 card deck with at least one heart and at least one club in the hand selected.
4. Let and be consecutive integers. SFTSOC that and are both even. That means and . So there is an integer such that and there is an integer such that . Let be the smaller of the consecutive integers, so . Which means . Since there are no such integers and , we have a contradiction ⇒⇐. Therefore and cannot both be even, which proves the proposition.
5. Suppose that and are sets. SFTSOC that Let . By definition of set difference, that means and . Let . By definition of set difference, that means and . Since and , and since and , then .
6. 1. Let , , and be sets and suppose A ⊆ B and B ⊆ C. SFTSOC that .
   2. Suppose and . SFTSOC that .
   3. Supposeand are two distinct circles. SFTSOC that and intersect at more than two points.
7. 1. There are ways to select 48 participants to be given a certificate of distinction out of the 509 participants.
   2. There are ways to select the top ten participants for trophies out of the 509 participants.
   3. There are ways to select the top ten participants for trophies out of the 509 participants. There are ways to select 38 more participants to be given a certificate of distinction out of the 499 participants left. So there are ways to select the top ten participants for trophies and then select 38 more to be given a certificate of distinction.